



# Cambridge International AS & A Level

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## FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

1 (a) Given that  $a$  is an integer, show that the system of equations

$$\begin{aligned} ax + 3y + z &= 14, \\ 2x + y + 3z &= 0, \\ -x + 2y - 5z &= 17, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

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(b) Find the value of  $a$  for which  $x = 1$ ,  $y = 4$ ,  $z = -2$  is the solution to the system of equations in part (a). [1]

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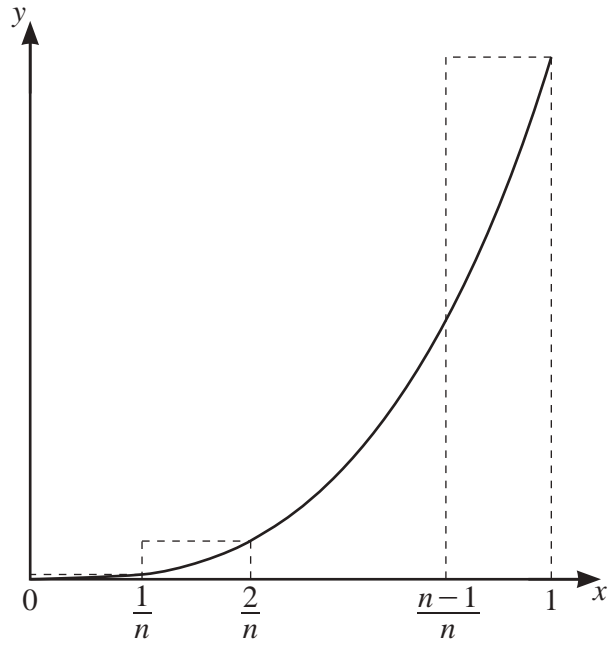
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3



The diagram shows the curve with equation  $y = x^3$  for  $0 \leq x \leq 1$ , together with a set of  $n$  rectangles of width  $\frac{1}{n}$ .

(a) By considering the sum of the areas of these rectangles, show that  $\int_0^1 x^3 dx < U_n$ , where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \quad [4]$$

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- (b) Use a similar method to find, in terms of  $n$ , a lower bound  $L_n$  for  $\int_0^1 x^3 dx$ . [4]

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- (c) Find the least value of  $n$  such that  $U_n - L_n < 10^{-3}$ . [2]

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**5 (a)** State the sum of the series  $z + z^2 + z^3 + \dots + z^n$ , for  $z \neq 1$ . [1]

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**(b)** Given that  $z$  is an  $n$ th root of unity and  $z \neq 1$ , deduce that  $1 + z + z^2 + \dots + z^{n-1} = 0$ . [2]

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**(c)** Given instead that  $z = \frac{1}{3}(\cos \theta + i \sin \theta)$ , use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} 3^{-m} \cos m\theta = \frac{3 \cos \theta - 1}{10 - 6 \cos \theta}. \quad [7]$$

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A series of horizontal dotted lines for writing.

6 The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) Find a matrix  $\mathbf{P}$  and a diagonal matrix  $\mathbf{D}$  such that  $\mathbf{A}^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ . [7]

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- (b) Use the characteristic equation of  $\mathbf{A}$  to find  $\mathbf{A}^3$ . [4]
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7 (a) It is given that  $y = \operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$ .

Express  $\cosh y$  in terms of  $x$  and hence show that  $\sinh y \frac{dy}{dx} = -\frac{1}{\left(x + \frac{1}{2}\right)^2}$ . [3]

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(b) Find the first three terms in the Maclaurin's series for  $\operatorname{sech}^{-1}\left(x + \frac{1}{2}\right)$  in the form

$$\ln a + bx + cx^2,$$

where  $a$ ,  $b$  and  $c$  are constants to be determined. [7]

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8 The curve  $C$  has parametric equations

$$x = 2 \cosh t, \quad y = \frac{3}{2}t - \frac{1}{4} \sinh 2t, \quad \text{for } 0 \leq t \leq 1.$$

(a) Find  $\frac{dx}{dt}$  and show that  $\frac{dy}{dt} = 1 - \sinh^2 t$ . [3]

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The area of the surface generated when  $C$  is rotated through  $2\pi$  radians about the  $x$ -axis is denoted by  $A$ .

(b) (i) Show that  $A = \pi \int_0^1 \left(\frac{3}{2}t - \frac{1}{4} \sinh 2t\right)(1 + \cosh 2t) dt$ . [4]

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(ii) Hence find  $A$  in terms of  $\pi$ ,  $\sinh 2$  and  $\cosh 2$ .

[6]

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